* NATIONAL RESEARCH FELLOW.

[1] M. Born and W. Heisenberg, *Ann. d. Physik*, **74**, 1, 1924.

[2] K. Schwarzschild, *Ber. d. Berliner Akad.*, **1916**, p. 548.

[3] P. S. Epstein, *Verh. d. Deutsch Phys. Ges.*, **18**, 398, 1916; *Phys. Zeits.*, **20**, 289, 1919.

[4] F. Reiche, *Phys. Zeits.*, **19**, 394, 1918.

[5] H. A. Kramers, *Zeits. f. Physik*, **13**, 343, 1923.

[6] When the work here presented was done, there was no published work dealing with this problem, except a first approximation by Reiche (loc. cit.) for the case when the asymmetry is small. Quite recently, however, F. Lütgemeier (*Zeits. f. Physik*, **38**, 251, 1926) has published a solution of it. The present mode of treatment and the resulting formula for W are different from his. The formula for W here derived contains only even powers of the quantum numbers, whereas his contain both even and odd powers. While his two formulas are doubtless equivalent to the one developed here, the differences referred to seemed to justify the publication of our results.

THE DIELECTRIC CONSTANT OF SYMMETRICAL POLYATOMIC DIPOLE-GASES ON THE NEW QUANTUM MECHANICS

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In a recent note in these PROCEEDINGS[1] the problem of the dielectric constant of diatomic dipole-gases has been discussed from the viewpoint of the new quantum mechanics. In the meantime a paper by Dennison[2] has appeared, in which frequencies and amplitudes are computed for the more general system of a rigid polyatomic molecule possessing an axis of symmetry and hence two equal moments of inertia. It is, therefore, possible now to deal also with the question of the dielectric constant of such a molecule with a permanent electric moment μ. The problem is simplified if it be remembered that due to the symmetry the electric moment will lie along the axis of the molecule. An example is probably NH_3 with the three H-nuclei at the corners of an equilateral triangle forming the base of a symmetrical pyramid at the apex of which the N-nucleus is located.

Following Dennison's notation we shall call the moment of inertia about the axis of figure C, the other one A. In his paper, to which we must refer concerning all details, it is shown that the system under consideration is characterized by three quantum numbers j, n, m (there called m, n, σ), if a uniform precession be superimposed on the body to remove the degeneracy of spacial orientation. The first of these determines the total moment of momentum, the second the moment of momentum about the axis of symmetry, while the last represents in units $h/2\pi$ the moment of momentum along the axis of precession z. If the energy differences due to the precession be neglected, the energy of the state j, n, m is given by

$$W^0_{j,n} = \left[\frac{1}{A} j(j+1) + \left(\frac{1}{C} - \frac{1}{A}\right)n^2 + \text{const.}\right]\frac{h^2}{8\pi^2}. \qquad (1)$$

In order that this expression for $C = 0$, $n = 0$ may become identical in its dependence on j with that for the simple rotator in space, j must have integral values. For a given value of j, n and m may have all integral values obeying the restrictions

$$|n| \leqq j, \qquad |m| \leqq j.$$

Since the electrical moment lies along the axis of symmetry, the transitions involving the frequency of rotation about this axis have zero amplitudes; i.e., radiative transitions in which n changes do not occur. With this in mind it is seen from Dennison's results that in the vector matrix representing the electric moment of the moving system, the only terms different from zero are those corresponding to transitions $(j, n, m; j, n, m)$, $(j, n, m; j-1, n, m)$, $(j, n, m; j-1, n, m \pm 1)$, $(j, n, m; j, n, m-1)$. Of these again only the first two give rise to terms in the z-component $\mathbf{P}^0_z$ of the electrical moment. From (1) and the frequency condition we find their frequency to be

$$\nu(j, n; j-1, n) = -\nu(j-1, n; j, n) = \frac{h}{4\pi^2 A} j \qquad (2)$$

if again the precessional frequency be neglected. Furthermore the corresponding amplitudes are

$$\left.\begin{aligned} P^0_z(j, n, m; j, n, m) &= \frac{nm}{j(j+1)}\,\mu, \\ P^0_z(j, n, m; j-1, n, m) &= \sqrt{\frac{(j^2-n^2)(j^2-m^2)}{j^2(2j-1)(2j+1)}}\,\mu. \end{aligned}\right\} \qquad (3)$$

The procedure to find the dielectric constant due to the permanent moments of the molecules is entirely analogous to that used in the case of diatomic molecules. From the perturbation theory[3] it follows that if a constant electric field E is applied parallel to z, the matrix $\mathbf{P}_z$ is now given in first approximation by

$$\mathbf{P}_z = \mathbf{P}^0_z + \mathbf{P}^1_z$$

and hence the terms independent of the time $P_z(j, n, m; j, n, m)$ by

$$P_z(j, n, m; j, n, m) = P^0_z(j, n, m; j, n, m) + P^1_z(j, n, m; j, n, m), \qquad (4)$$

where

$$P_z^1(j, n, m;\ j, n, m) = -\frac{2E}{h}\sum_{j', n', m'}{}' \frac{P_z^0(j, n, m;\ j', n', m')\ P_z^0(j', n', m';\ j, n, m)}{\nu(j, n, m;\ j', n', m')}, \tag{5}$$

the accent at the summation sign indicating that j', n', m' must not have the values j, n, m simultaneously. Similarly the energy values of the stationary states of the perturbed system are in first approximation

$$W_{j,n,m} = W_{j,n}^0 + W_{j,n,m}^1, \tag{6}$$

where

$$W_{j,n,m}^1 = -E\ P_z^0(j, n, m;\ j, n, m). \tag{7}$$

By introducing (2) and (3) in (4) and (5) we get

$$P_z(j, n, m;\ j, n, m) = \frac{nm}{j(j+1)}\mu - \frac{8\pi^2 AE\mu^2}{h^2}\left\{\frac{(j^2 - n^2)(j^2 - m^2)}{j^3(2j-1)(2j+1)} - \frac{[(j+1)^2 - n^2][(j+1)^2 - m^2]}{(j+1)^3(2j+1)(2j+3)}\right\}.$$

The probability of an atom being in the state j, n, m is

$$\frac{e^{-W_{j,n,m}/kT}}{\sum_{j,n,m} e^{-W_{j,n,m}/kT}},$$

and the dielectric constant ϵ due to the permanent moments of the molecules is then given by

$$\frac{3}{4\pi}\frac{\epsilon - 1}{\epsilon + 2} = \frac{N\sum_{j,n,m} P_z(j, n, m;\ j, n, m)e^{-W_{j,n,m}/kT}}{E\sum_{j,n,m} e^{-W_{j,n,m}/kT}}, \tag{8}$$

where N is the number of molecules per cm.[3] The sum in the numerator we wish to compute to terms linear in E. We may hence write for it with the aid of (4) and (6)

$$\sum_{j,n,m} [P_z^0(j, n, m;\ j, n, m) + P_z^1(j, n, m;\ j, n, m)]e^{-(W_{j,n}^0 + W_{j,n,m}^1)/kT}$$

$$= \sum_{j,n,m} [P_z^0(j, n, m; j, n, m) + P_z^1(j, n, m; j, n, m)]\left(1 - \frac{W_{j,n,m}^1}{kT}\right)e^{-W_{j,n}^0/kT}$$

$$= \sum_{j,n,m} [P_z^0(j, n, m;\ j, n, m) - P_z^0(j, n, m;\ j, n, m)\frac{W_{j,n,m}^1}{kT} + P_z^1(j, n, m;\ j, n, m)]e^{-W_{j,n}^0/kT}.$$

Substituting for P_z^0, P_z^1 and W^1 their values from (3), (5) and (7), and summing over m from $-j$ to j, we get zero from the first term while the other two give

$$\frac{E\mu^2}{3kT}\sum_{j,n}\left[\frac{n^2(2j+1)}{j(j+1)}+\frac{1}{\alpha}\frac{n^2(2j+1)}{j^2(j+1)^2}\right]e^{-\alpha j(j+1)-\beta n^2} \tag{9}$$

if we introduce the abbreviations

$$\alpha = \frac{h^2}{8\pi^2 kT}\frac{1}{A}, \qquad \beta = \frac{h^2}{8\pi^2 kT}\left(\frac{1}{C}-\frac{1}{A}\right).$$

In the sum in the denominator of (8) we may put $W^0_{j,n}$ instead of $W_{j,n,m}$ and get hence

$$\sum_{j,n}(2j+1)e^{-\alpha j(j+1)-\beta n^2}. \tag{10}$$

The two sums (9) and (10) we may approximate by integrals if α and β are small compared to unity, i.e., if T is sufficiently large. They become, respectively,

$$\frac{2E\mu^2}{3kT}\int_0^\infty dn.n^2e^{-\beta n^2}\int_n^\infty dj.(2j+1)\left[\frac{1}{j(j+1)}+\frac{1}{\alpha j^2(j+1)^2}\right]e^{-\alpha j(j+1)} \tag{11}$$

$$2\int_0^\infty dn.e^{-\beta n^2}\int_n^\infty dj.(2j+1)e^{-\alpha j(j+1)}. \tag{12}$$

The integrations with respect to j can be performed if we introduce as new variable $\alpha j(j+1) = x$. Then we have in the first integral from integration by parts

$$\int_n^\infty dj.(2j+1)\left[\frac{1}{j(j+1)}+\frac{1}{\alpha j^2(j+1)^2}\right]e^{-\alpha j(j+1)}$$
$$= \int_{\alpha n(n+1)}^\infty dx.\left(\frac{1}{x}+\frac{1}{x^2}\right)e^{-x} = -\frac{1}{x}e^{-x}\Big]_{\alpha n(n+1)}^\infty = \frac{1}{\alpha n(n+1)}e^{-\alpha n(n+1)},$$

in the second

$$\int_n^\infty dj.(2j+1)e^{-\alpha j(j+1)} = \frac{1}{\alpha}\int_{\alpha n(n+1)}^\infty dx.e^{-x} = \frac{1}{\alpha}e^{-\alpha n(n+1)}.$$

The first expression (11) thus becomes

$$\frac{2E\mu^2}{3kT\alpha}\int_0^\infty dn.\frac{n}{n+1}e^{-\alpha n(n+1)-\beta n^2},$$

the expression (12)

$$\frac{2}{\alpha}\int_0^\infty dn.e^{-\alpha n(n+1)-\beta n^2}.$$

In the former $n/(n + 1)$ may be replaced without error by unity in the approximation we are interested in. Substituting the values so obtained for the sums in (8) gives

$$\frac{3}{4\pi}\frac{\epsilon - 1}{\epsilon + 2} = \frac{N\mu^2}{3kT}.$$

We see then that here, just as in the case of diatomic molecules, the permanent moment of the molecules and the part of the dielectric constant due to it are related by Debye's equation derived on the classical theory provided the temperature is sufficiently high.

As regards the behavior of the bands emitted by dipole-molecules of the kind considered in an electric field, there should be a linear Stark effect for all lines arising from transitions between states for which n is different from zero, as may be seen directly from (7) and (3). The deflection of the molecules in an electric Stern-Gerlach experiment of the kind described in the previous paper[1] can be easily predicted, too, by introducing the value of P determined by equations (4), (3) and (5) into the formula for the deflection.

[1] R. de L. Kronig, *Proc. Nat. Acad. Sci.*, **12**, 488, 1926. See also L. Mensing and W. Pauli, *Phys. Zs.*, **27**, 509, 1926; J. H. Van Vleck, *Nature*, **118**, 226, 1926.

[2] D. M. Dennison, *Phys. Rev.*, **28**, 318, 1926.

[3] M. Born, W. Heisenberg and P. Jordan, *Zs. f. Phys.*, **35**, 557, 1926.

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*TROPISMS OF MAMMALS**

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Conceptions of the mechanism of adjustor function in the central nervous system have chiefly been based upon results of "learning" tests, as in a maze, and upon the method of conditioned reflexes. These modes of experimentation are cumbersome, and do not give direct approach to the understanding of variability in conduct and the basis of moment-to-moment adjustment in behavior. On the other hand, interpretation in terms of reflexes has obvious limitations.

We desired to see if certain conceptions of the analysis of conduct, as derived from study of lower animals, might not after all be applicable in this connection, and to obtain if possible another route of approach; one which would permit the expression of behavior in quantitative terms and make possible the dynamical formulation of conduct in situations